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Deformation of the Geomagnetic Field by the Solar Wind

GILBERT D. MEAD

CODE NONE

Goddard Space Flight Center, Greenbelt, Maryland

Abstract. From the three-dimensional numerical solution to the Chapman-Ferraro problem of a steady solar wind perpendicularly incident upon a dipole field, a simple spherical harmonic description of the distorted field is obtained. From this, a three-dimensional picture of the field line configuration within the magnetosphere is given. The field lines are found to be compressed on both the daytime and nighttime sides. The behavior of the field lines on the daylight side changes abruptly as one approaches a critical latitude, which ranges between 80° and 85°, depending on the intensity of the solar wind. Above this latitude, lines originating along the noon meridian pass over the north pole and cross the equator along the midnight meridian. The behavior of conjugate-point phenomena and trapped particles near this critical latitude is discussed. Magnetic changes at the earth's surface due to an increase in the solar wind intensity are calculated. The diurnal variations due to a steady solar wind are also calculated; they are found to be small compared with observed S_q fields.

INTRODUCTION

This paper will discuss a number of geophysical phenomena related to the presence of the solar wind and the resulting distortion of the geomagnetic field. A number of calculations will be made using a description of the distorted field given by an expansion in spherical harmonics. We shall consider the field to consist of two parts: an internal part represented by a simple dipole colinear with the earth's axis of rotation, plus an external part due to surface currents on the boundary of the magnetosphere. The numerical values of the spherical harmonic coefficients describing the external part are obtained from a three-dimensional numerical solution to the Chapman-Ferraro problem. This problem consists of determining the shape of the boundary surface between a steady, zero-temperature, field-free plasma perpendicularly incident upon a dipole field, where the plasma is assumed to be specularly reflected from the surface. Once the shape is determined, the perturbation field at any point due to the surface currents can be calculated from the Biot-Savart law by performing a surface integration over the boundary. An accompanying paper [*Mead and Beard*, 1964] gives the results of the calculations on the shape of the boundary and shows how this solution was obtained.

The most common method for describing the geomagnetic field is to express it as the negative gradient of a scalar potential. This potential is given as the sum of a series of spherical harmonic functions with arbitrary coefficients. The values of the coefficients are usually determined by making an analysis of one or more components of the field at a large number of points on the surface of the earth. Such analyses have usually shown that only about 1 per cent or less of the surface magnetic field can be of origin external to the earth. Near the earth, therefore, terms in the spherical harmonic expansion due to external sources can usually be neglected.

At large distances from the earth, however, the part of the field due to external sources becomes proportionately much larger, because terms in the magnetic field description associated with internal sources fall off as $1/r^3$ or faster, where r is the distance from the origin. On the other hand the field associated with external source terms is either constant or proportional to positive powers of r.

For calculations of the magnetic field at large distances, therefore, it is important to have some knowledge of the external source terms. These sources fall into three main groups: ionosphere currents, ring currents due to trapped radiation, and currents due to the presence of the solar wind at the outer boundary of the magnetosphere.

Ionosphere currents are the source of most of the daily variations in magnetic field elements observed at the surface of the earth. But outside the ionosphere these currents become internal sources, and therefore the associated field falls off rapidly along with the main field. Thus they do not contribute much to the field at large distances.

Ring currents associated with the longitudinal drift of radiation-belt particles have long been postulated as the current source producing the main phase of magnetic storms. *Akasofu* [1963] has recently reviewed the subject of ring currents. The location and intensity of these currents have not yet been firmly established, however, and without such knowledge it is difficult to include them in any calculation of magnetic fields in space.

This paper is concerned with the effects of the third source, currents at the boundary of the magnetosphere. From a spherical harmonic description of the distorted field, a three-dimensional picture of the field lines is presented. A number of interesting results emerging from this picture are discussed. In particular, the shape of the lines and the magnitude of the field along the lines in the polar region bear upon the trapping of charged particles in this region. The behavior of conjugate-point phenomena also changes rapidly in this region.

By varying the intensity of the solar wind, the changes in the magnetic field components at varying positions on the surface of the earth can be calculated. These can then be related to the sudden commencement phase of magnetic storms, which is usually considered to be due to an increase in the solar wind. And, finally, the daily variation in the earth's field at various latitudes due to the presence of a steady solar wind is calculated and compared with the observed S_q (solar quiet) fields.

Spherical Harmonic Description of the Distorted Field

Definitions. We wish to express the geomagnetic potential in terms of sources within the earth plus current sources at the boundary of the magnetosphere. The choice of azimuthal angle, however, is different for these two sources. The terms in the expansion due to internal sources depend on geographic longitude, whereas the surface current fields will be assumed to depend only on the position of the sun, i.e., local time. If it is assumed that the region between the earth's surface and the magnetosphere boundary is source free—that is, if ionosphere currents and ring currents are neglected—the total geomagnetic potential V_T within this region may be expressed in terms of a spherical harmonic series

$$V_T = a \sum_{n=1}^{\infty} \left[\left(\frac{a}{r}\right)^{n+1} T_n(\theta, \alpha) + \left(\frac{r}{a}\right)^n \bar{T}_n(\theta, \phi) \right]$$
$$= V(r, \theta, \alpha) + \bar{V}(r, \theta, \phi) \qquad (1)$$

where a is the mean radius of the earth, r the distance from the earth's center, θ the colatitude, α the geographic east longitude, and ϕ the local time measured from the *midnight* meridian. Throughout this paper barred quantities are quantities related to external sources.

Here

$$T_n = \sum_{m=0}^{n} (g_n^m \cos m\alpha + h_n^m \sin m\alpha) P_n^m(\cos\theta) \qquad (2)$$

and

$$\bar{T}_n = \sum_{m=0}^{n} (\bar{g}_n^m \cos m\phi + \bar{h}_n^m \sin m\phi) P_n^m(\cos\theta) \qquad (3)$$

where

$$P_n^m(\cos\theta) = \left[2 \frac{(n-m)!}{(n+m)!} \right]^{1/2} P_{n,m}(\cos\theta), \qquad m > 0 \qquad (4)$$

$$P_n^m(\cos\theta) = P_{n,m}(\cos\theta) \qquad m = 0 \qquad (5)$$

and

$$P_{n,m}(x) = \frac{1}{2^n n!} (1 - x^2)^{m/2} \frac{d^{n+m}}{dx^{n+m}} (x^2 - 1)^n \qquad (6)$$

Note that the associated Legendre functions used in this paper (P_n^m) are those with the Schmidt normalization, as has been the convention in geomagnetic applications.

The three components of the magnetic field at all points inside the magnetosphere are then given by taking the negative gradient of V_T,

$$B_r = -\partial V_T/\partial r = -Z \qquad (7)$$

$$B_\theta = -(1/r)(\partial V_T/\partial \theta) = -X \qquad (8)$$

$$B\phi = -\frac{1}{r \sin\theta} \left(\frac{\partial V}{\partial \alpha} + \frac{\partial \bar{V}}{\partial \phi} \right) = Y \qquad (9)$$

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where X, Y, and Z are the north, east, and downward vertical components of the field, respectively.

For the calculations in this paper, certain simplifications will be made. First, the internal earth's field is assumed to be a pure dipole whose axis is colinear with the axis of rotation. This is equivalent to setting all internal coefficients to zero except g_1^0, which is set equal to 0.31 gauss. Second, the direction of the solar wind is assumed to be perpendicular to the dipole axis and parallel to the earth-sun line. With these assumptions the magnetosphere and its associated field have two planes of symmetry, the equator and the noon-midnight meridian. In particular, B_r and B_ϕ are antisymmetric about the equator, and B_ϕ is also antisymmetric about the noon meridian. B_r and B_θ are symmetric about the noon meridian, and B_θ is also symmetric about the equator. Mathematically, this is equivalent to setting three-quarters of the external coefficients equal to zero. In particular, all the $\bar{h}$'s vanish, and only the $\bar{g}$'s for which $n + m$ is odd remain.

If terms only through $n = 2$ are taken, the north, east, and vertical components of the total magnetosphere field are

$$X = -B_\theta = \frac{0.31 \sin \theta}{r^3} - \bar{g}_1{}^0 \sin \theta + \sqrt{3}\, \bar{g}_2{}^1 r(2 \cos^2 \theta - 1) \cos \phi \qquad (10)$$

$$Y = B_\phi = \sqrt{3}\, \bar{g}_2{}^1 r \cos \theta \sin \phi \qquad (11)$$

$$Z = -B_r = \frac{0.62 \cos \theta}{r^3} + \bar{g}_1{}^0 \cos \theta + 2\sqrt{3}\, \bar{g}_2{}^1 r \sin \theta \cos \theta \cos \phi \qquad (12)$$

where r is now measured in units of earth radii.

In the equatorial plane, only the X component is nonvanishing; it is given by

$$X_{eq} = \frac{0.31}{r^3} - \bar{g}_1{}^0 - \sqrt{3}\, \bar{g}_2{}^1 r \cos \phi \qquad (10')$$

In cartesian coordinates, with the x axis pointed toward the sun and the z axis toward the north star, the components through $n = 2$ *due to surface currents only* are

$$B_x = \sqrt{3}\, \bar{g}_2{}^1 z$$

$$B_y = 0$$

$$B_z = -\bar{g}_1{}^0 + \sqrt{3}\, \bar{g}_2{}^1 x$$

with x, y, and z measured in earth radii.

Note that the spherical harmonic expansion for the distorted field is good only out to the boundary of the magnetosphere. Outside that point, the surface current field is equal and opposite to the dipole field at every point, making the total field zero in the model used here.

Values of the coefficients. Numerical values for the coefficients $\bar{g}_n{}^m$ were obtained from the solution to the problem of a uniformly directed, zero-temperature solar wind perpendicularly incident upon a dipole field. The details of this solution and the shape of the boundary as determined by it are given in an accompanying paper [*Mead and Beard*, 1964]. To facilitate the calculations, a different orientation of the coordinate system was used in that paper, with the polar axis pointed toward the sun instead of the north star. *The angles θ and ϕ therefore have different meanings in the two papers.* In making the calculations for this paper, the appropriate factors were used to transform quantities into the rotated coordinate system.

The vector field due to the surface currents on the boundary was calculated at 273 points inside the magnetosphere, using equation 16 of *Mead and Beard* [1964] to perform the surface integrals. About half the points were distributed more or less uniformly within the solar side. The remainder were on the dark side, some points being out as far as twice the solar-side-boundary distance. (Since the calculations of the magnetosphere assumed a zero-temperature solar wind, the cavity extends to infinity on the dark side without closing.) A least-squares fitting program was used to determine the best values of all coefficients up to a given value of n. The resulting coefficients for $n_{max} = 6$ are given in Table 1.

The strength of the perturbed field must, of course, depend on the intensity of the solar wind. This dependence is expressed in the coefficients through the parameter r_b, the distance from the center of the earth to the boundary along the earth-sun line. *Mead and Beard* [1964] find that this distance is related to the solar wind parameters by

$$r_b = 1.068(M^2/4\pi mnv^2)^{1/6} \qquad (13)$$

where M is the earth's dipole moment and $2mnv^2$ is the pressure of the solar wind on the magnetosphere at the subsolar point.

The coefficients in Table 1 approximate the

TABLE 1. Coefficients in the Spherical Harmonic Expansion To Be Used in Equation 3

The parameter r_b is the distance to the boundary along the earth-sun line expressed in earth radii. Fields are in gauss.

n, m	$\bar{g}_n^m r_b^{n+2}$	n, m	$\bar{g}_n^m r_b^{n+2}$
1, 0	−0.2511	5, 0	0.0057
2, 1	0.1242	5, 2	−0.0108
3, 0	−0.0072	5, 4	−0.0010
3, 2	−0.0233	6, 1	−0.0013
4, 1	0.0240	6, 3	0.0019
4, 3	0.0016	6, 5	0.0004

three calculated components of the field at the 273 selected points with a root mean square deviation of 0.3 gamma (assuming $r_b = 10$ earth radii), or about 1 per cent of the average perturbed field. ($1\ \gamma = 10^{-5}$ gauss.)

The two most important coefficients are $\bar{g}_1^0$, which gives a constant field directed parallel to the dipole, and $\bar{g}_2^1$, which gives a constant field gradient along the earth-sun line, i.e., a linear field. All other coefficients give fields down by an order of magnitude or more in regions not too far from the earth. It would be useful for many calculations, therefore, to have a simple description of the distorted field based on using these first two coefficients in equations 10–12. A least-squares fit with two coefficients was therefore made to 224 points lying in the region out to 0.7 r_b. The resulting coefficients were

$$\bar{g}_1^0 = -0.2515/r_b^3 \quad \text{gauss}$$

$$\bar{g}_2^1 = 0.1215/r_b^4 \quad \text{gauss}$$

with r_b expressed in earth radii.

These coefficients approximate the field in this region with a root mean square deviation of 1.0 gamma, or about 3 per cent of the perturbed field. This field description is almost as simple as the image dipole description used by many in the past to make calculations on the effects of the solar wind, and we believe it to be much more accurate than the image dipole. Such a description reproduces with only slight changes all the geophysical effects calculated and described below.

Very near the earth, terms proportional to g_1^0 provide the main contribution to the surface current field. Using (13) in (10)–(12), with $M/a^3 = 0.31$ gauss and $\bar{g}_2^1 = 0$, an approximate value for the surface current field at or near the earth's surface directly in terms of solar wind parameters is:

$$X = 0.0305\sqrt{nv}\sin\theta \qquad (10a)$$

$$Y = 0 \qquad (11a)$$

$$Z = -0.0305\sqrt{nv}\cos\theta \qquad (12a)$$

or

$$B_s = 0.0305\sqrt{nv}$$

with n in protons/cm^3 and v in km/sec, and the field in gammas. If in addition to the protons there is a component of alpha particles with density n_α, the square root terms becomes $\sqrt{n_p + 4n_\alpha}$. Thus, a solar wind with velocity 500 km/sec, containing 2 protons/cm^3 and 0.2 alpha/cm^3, would produce a surface current field of 25.5 gammas at the earth. The magnetopause subsolar point would be at 9.95 earth radii.

Midgley [1964] has also obtained a set of spherical harmonic coefficients to describe the perturbation of the geomagnetic field by the solar wind, based on the calculations of *Midgley and Davis* [1963]. By comparing his definition of the scalar potential (Midgley's equations 2 and 7) with ours (equations 1 and 3), we obtain the following relation between the two sets of coefficients:

$$\bar{g}_n^m/a^{n-1} = J_0 T_{nm}^0/r_0^{n-1}$$

for all n and m, with $J_0 \equiv (nmv^2/\pi)^{1/2}$ and $r_0 \equiv (M^2/4\pi nmv^2)^{1/6}$, so that $J_0 r_0^3 = M/2\pi$. Using (13),

$$\bar{g}_n^m\left(\frac{r_0}{a}\right)^{n+2} = \frac{1}{(1.068)^{n+2}}\left[\bar{g}_n^m\left(\frac{r_b}{a}\right)^{n+2}\right]$$

$$= \frac{1}{2\pi}\frac{M}{a^3}T_{nm}^0 \qquad (14)$$

with $M/a^3 = 0.31$ gauss. Since our Table 1 gives the quantity in the brackets, these coefficients must be renormalized by the factor $(1/1.068)^{n+2}$ in order to compare them directly with Midgley's. Our renormalized coefficients are shown along with his in Table 4 of *Midgley* [1964]. The two most important coefficients, $\bar{g}_1^0$ and $\bar{g}_2^1$, agree within 4 per cent. All except the smallest of the remaining coefficients through $n = 6$ agree in sign and roughly in magnitude. The fact that these results of the two calculations agree so well, in spite of the two completely different

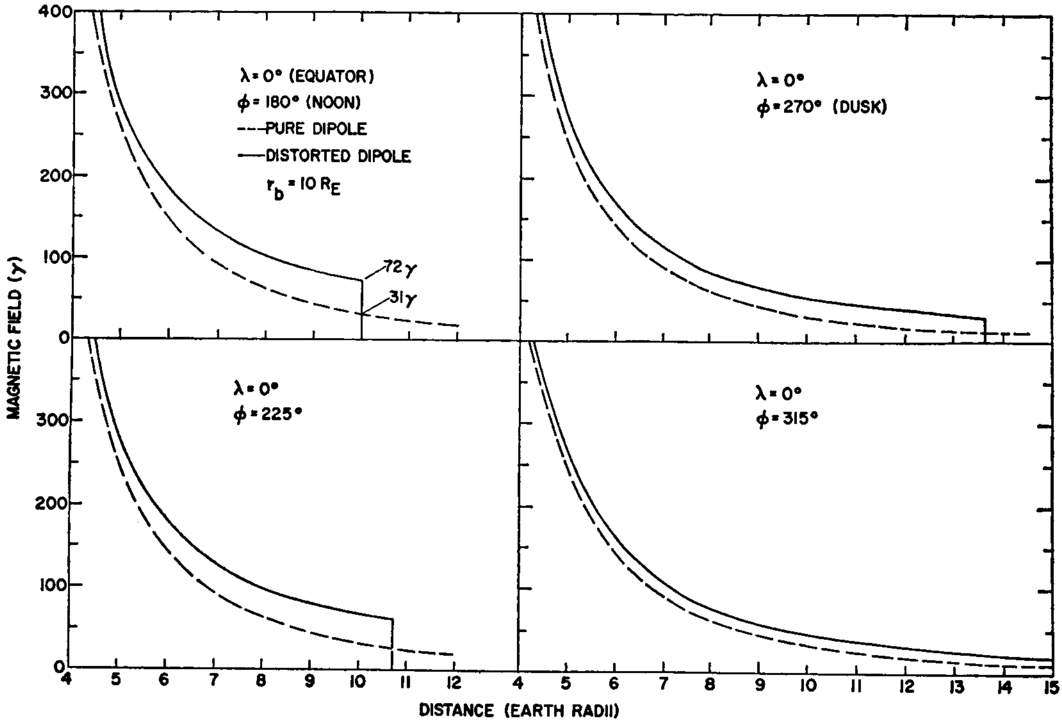

Fig. 1. The magnitude of the distorted field as compared with the undisturbed dipole field for several values of local time at the equator. The direction is everywhere perpendicular to the equatorial plane, which is considered to be parallel to the solar ecliptic plane in these calculations.

approaches to the problem, indicates that both solutions must be substantially correct.

Magnetic field due to surface currents. If $r_b = 10$ earth radii, a typical value observed by Explorer 12 [*Cahill and Amazeen*, 1963], the field due to surface currents is 25 gammas in the vicinity of the earth and 41 gammas just inside the boundary in the solar direction, in addition to the earth's field, and is essentially linear in between. Thus 58 per cent of the total field of 72 gammas at the subsolar point is produced by the surface currents.

These fields may be compared with those of *Beard and Jenkins* [1962], who calculated the surface current magnetic field in the noon-meridian plane. Assuming a hemispherical surface for the magnetosphere on the daylight side, they obtained a field of 14 gammas at the earth's surface and 35 gammas just inside the boundary in the solar direction when $r_b = 10$ earth radii, somewhat less than our values.

In Figures 1–3 the total field is compared with the undisturbed dipole field as a function of distance from the earth for several values of λ and ϕ, where λ is the latitude. Also shown are two angles δ and ϵ, which give the direction of the field. These are the same as the solar ecliptic angles, θ and ϕ, as defined in Figure 9 of *Heppner et al.* [1963] in their paper on the Explorer 10 results. δ is the angle between the direction of the magnetic field and the solar ecliptic plane, positive if directed north of the plane. ϵ is the angle between a vector pointed toward the sun and the component of the field in the ecliptic plane, measured counterclockwise when viewed from above. In the present calculations the ecliptic plane is identical with the earth's equatorial plane, since we have not considered any tilt in either the earth's axis or the earth's dipole moment with respect to the axis. The direction of the fields is plotted for the northern hemisphere and local times between noon and midnight. At corresponding southern latitudes, the angle δ is unchanged and ϵ is decreased or

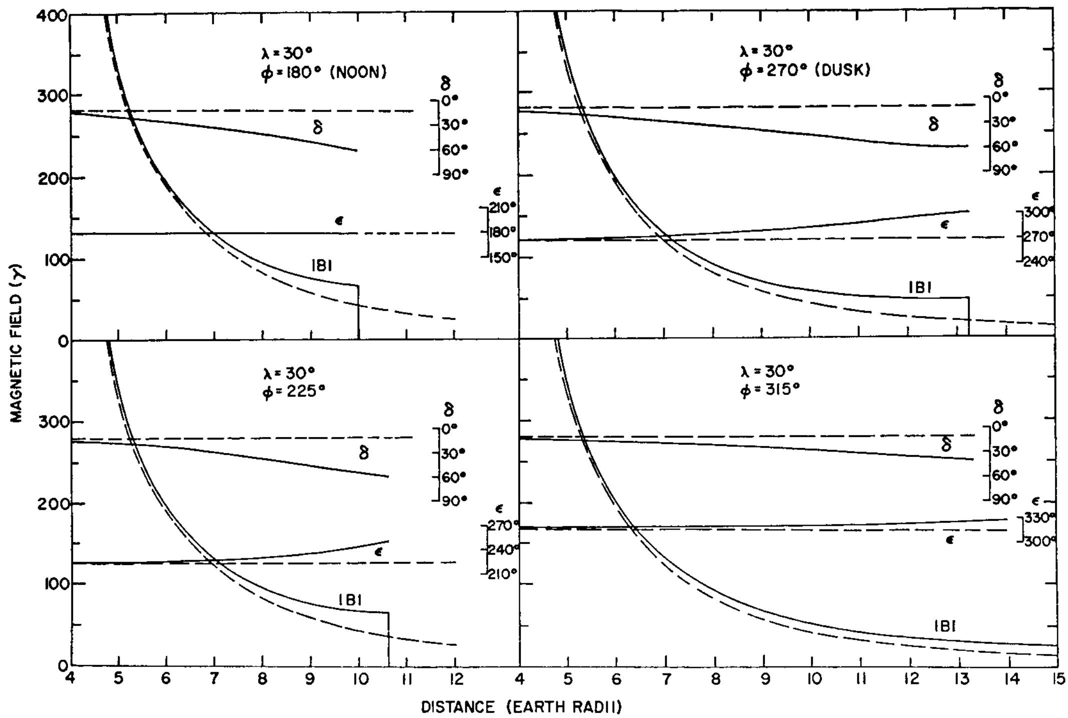

Fig. 2. The magnitude and direction of the distorted field and the dipole field at a latitude of 30°N. The angles δ and ϵ are the same as the solar ecliptic angles, θ and ϕ, as defined by *Heppner et al.* [1963].

increased by 180°. At points reflected about the noon meridian, δ is unchanged and ϵ must be replaced by 360° − ϵ. The magnitude of the field is unchanged.

At high latitudes the effect of the surface current field is to *reduce* the total field, and a smaller discontinuity exists when crossing the boundary. In some regions the magnitude of the field is essentially unchanged, but the direction changes significantly from that of a pure dipole. Only insignificant changes in either direction or magnitude occur at distances less than 4 or 5 earth radii if r_b = 10 earth radii.

Comparison with an image dipole. It is instructive to compare the surface current fields calculated here with those obtained by placing an image dipole of equal strength at a position 20 earth radii away, producing a plane boundary at 10 earth radii. Such a configuration has been used by several authors to simulate the magnetosphere surface current fields. The image dipole field would be 31 gammas at the subsolar boundary point, as compared with 41 gammas calculated here. Along the earth-sun line the image dipole field falls off very rapidly; it is 4 gammas in the vicinity of the earth and 1 gamma at 10 earth radii on the dark side, compared with 25 and 10 gammas, respectively, as calculated here. Thus the image dipole is grossly inaccurate in calculating effects near the earth due to magnetosphere surface currents. A modified image dipole, as used by *Hones* [1963], gives much more reasonable results.

Field Line Configuration within the Cavity

The description of the distorted dipole field as given in the previous section was used in making a number of field line calculations. The calculations were performed by a numerical integration of the following equations to obtain r, θ, and ϕ as a function of the parameter s, the distance along the field line:

$$dr/ds = B_r/B \tag{15}$$

$$d\theta/ds = (1/r)(B_\theta/B) \tag{16}$$

$$d\phi/ds = (1/r \sin \theta)(B_\phi/B) \tag{17}$$

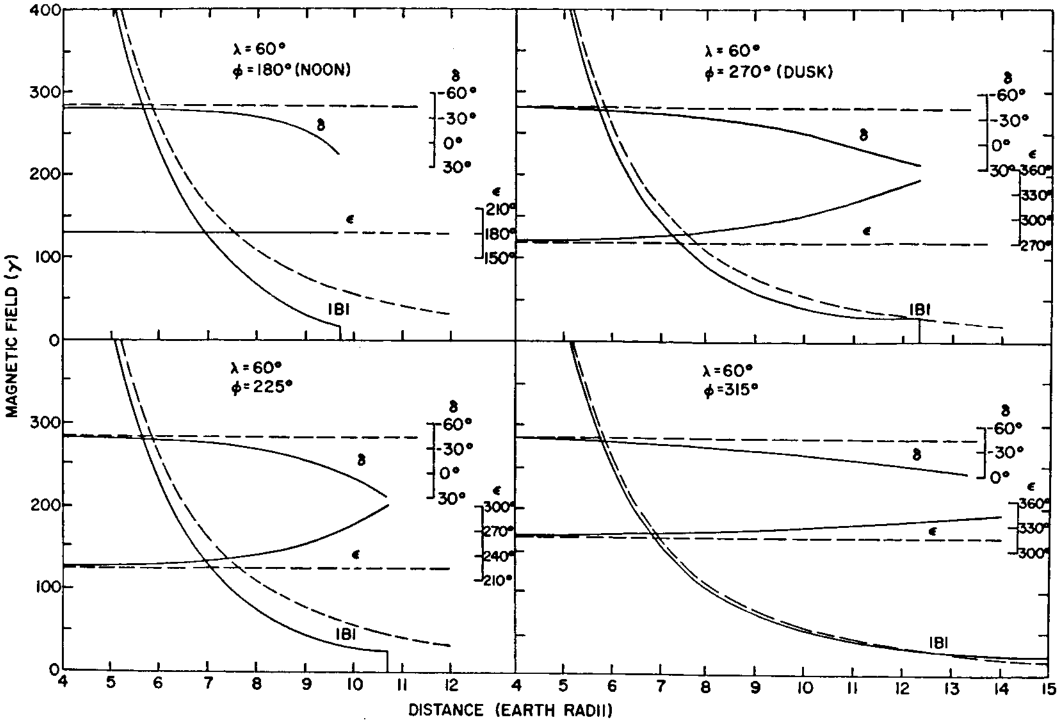

Fig. 3. The magnitude and direction of the field at a latitude of 60°N.

where B is the magnitude of the field.

The field line configuration in the noon-midnight meridian plane for $r_b = 10$ earth radii is shown in Figure 4. The field lines corresponding to a pure dipolar field are shown as dashed lines. This figure illustrates a number of interesting results:

1. As expected, the field lines at large distances are compressed on the sunlit side. Note, however, that the field lines on the dark side are also compressed, although not as much. It has sometimes been assumed that the field lines on the dark side would be extended farther out rather than compressed, but the present analysis does not support that assumption. The effect of compression is shown more clearly in Figure 5, where the equatorial crossing point of the field line in the presence of the solar wind is plotted as a function of the undisturbed position of the field line, for lines emanating at noon, dusk, and midnight local time. Note that the effect of distortion is evident only for field lines emanating from a latitude greater than 60°, corresponding to $L > 4$, where L is the usual McIlwain parameter.

2. On approaching the north pole along the noon meridian a critical latitude is reached beyond which all field lines emanating from the surface of the earth are bent back over the north pole and cross the equator along the midnight meridian; then they pass underneath the south pole and enter the earth again at the normal conjugate point.

3. The critical latitude at which this transition occurs is about 83°. If the intensity of the solar wind is reduced to make $r_b = 15$ earth radii, the transition occurs between 84° and 85°. If the intensity is increased, making $r_b = 5$ earth radii, the transition is between 80° and 81°. Thus the critical latitude is not very sensitive to the intensity of the solar wind.

4. Field lines emanating from the earth close to the critical latitude pass very near the so-called 'null point,' which separates field lines crossing the equator along the noon meridian from those crossing along the midnight meridian. At the null point, the magnitude of the field is zero and the boundary of the magnetosphere is tangential to the direction of the solar wind. From the field description given here the

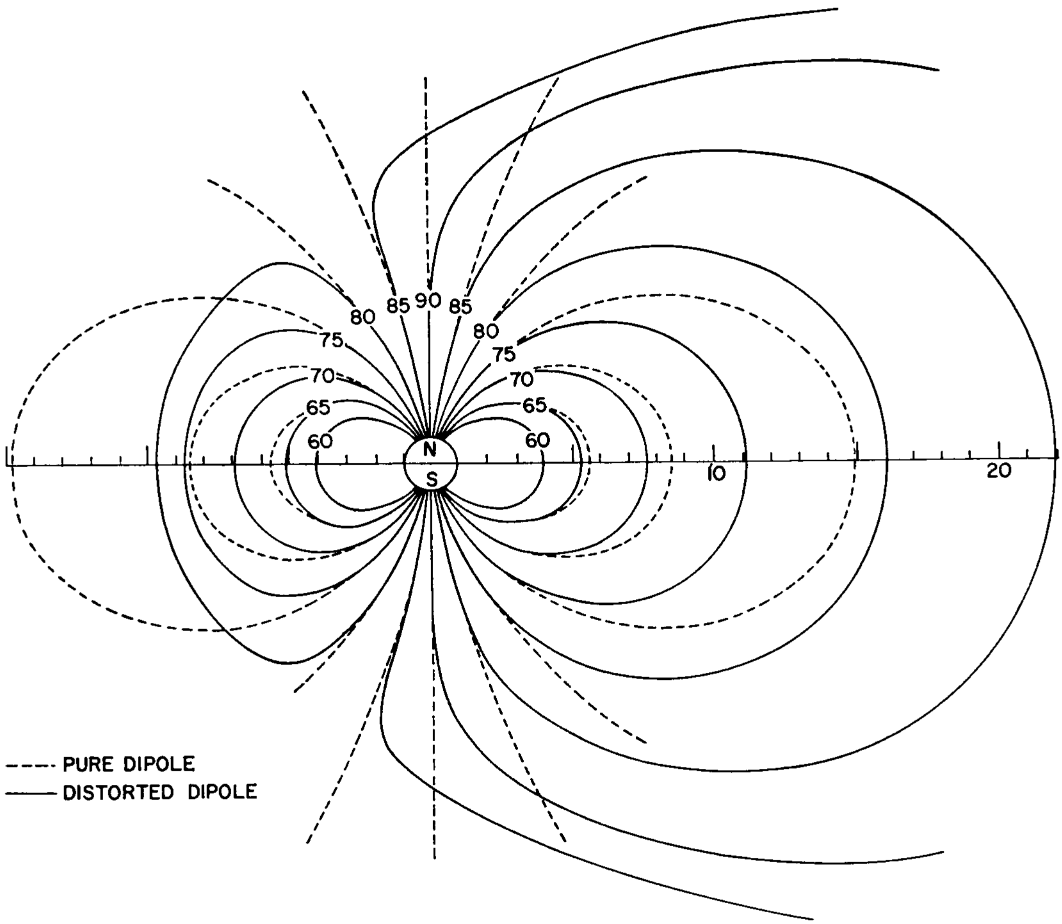

Fig. 4. Field line configuration in the noon-midnight meridian plane. With $r_b = 10$ earth radii, the critical latitude is about 83°. The dipole lines are compressed on both the daytime and nighttime side.

latitude of the null point is about 70°, independent of the intensity of the solar wind. The distance from the center of the earth to the null point is given by

$$r_n = 0.93r_b$$

5. The field lines very near the null point pass just inside the boundary of the geomagnetic cavity. This provides a way of checking the internal consistency of the method for deriving the fields: the boundary of the magnetosphere can be traced by plotting the loci of the 82° field line and the 84° field line as the azimuthal angle is varied. *Hones* [1963] has determined a magnetosphere boundary in this way, using a modified image dipole instead of a spherical harmonic field description. His results are qualitatively similar to ours. The method followed here is indeed internally consistent; the boundary as determined this way is nearly the same as that determined in the solution to the magnetosphere problem [*Mead and Beard,* 1964], except that the distant antisolar regions cannot be traced out in this fashion.

6. Since the distorted field used here is symmetric about the equatorial plane, pairs of conjugate points will always have the same longitude and equal but opposite latitude. However, the nature of the conjugacy at latitudes less than the critical latitude is much different from that in polar regions, since the polar lines of force travel out to much greater altitudes and are always on the dark side of the earth. Since the field in these regions is very weak and

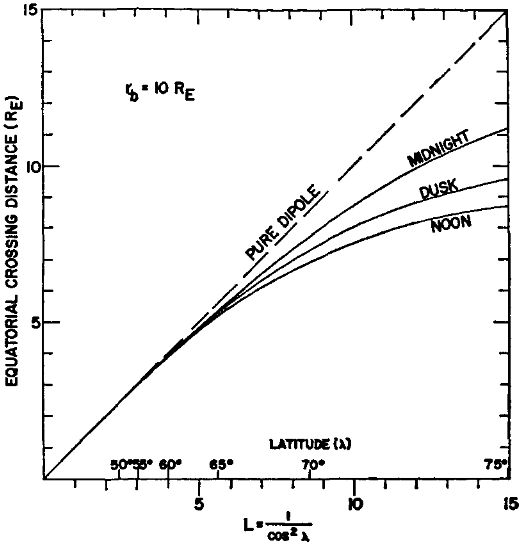

Fig. 5. Compression of field lines at the equator. $1/\cos^2 \lambda$ is the normal equatorial crossing radius for a pure dipole, and would correspond to McIlwain's L parameter in the absence of a solar wind.

turbulent, above the critical latitude much less correlation would be expected between geomagnetic phenomena at conjugate points. In addition, above the critical latitude, lines of force emanating from the night side of the earth are shorter than those that emanate from the day side but cross the equator on the night side. Thus at high latitudes correlations between phenomena at conjugate points should be greater at night than during the day. *Wescott and Mather* [1963] have examined magnetograms from high-latitude conjugate stations and have found just such a day-night effect. In this case the geomagnetic latitudes of the conjugate stations were 78.3°N and 79.0°S, somewhat less than the critical latitude found here. However, a day-night difference might still be expected, since the lines of force on the day side pass much closer to the null point, where the field is expected to be very weak and turbulent. Additional evidence for a sudden change in the nature of magnetic activity as critical latitude is approached has been given by *Lebeau and Schlich* [1962], who compared magnetograms from two nearby stations at geomagnetic latitudes 75.6°S and 78.3°S. They found that both stations experienced their maximum activity at midday, when the lines of force would pass closest to the null point. The correlation between average activities at the two stations was highest at midnight (≈0.95) and lowest at midday (≈0.65). Wescott and Mather, and Lebeau and Schlich explained their results with arguments similar to those presented here.

7. With the description of the distorted field used here, the magnitude of the field can be calculated everywhere along a field line. The results of such calculations are shown in Figures 6 and 7. Figure 6 shows the field along lines emanating from various latitudes on the midnight meridian. The numbers are not much different from those calculated from a pure dipole field. But Figure 7, showing lines along the noon meridian, indicates an unusual behavior setting in as the critical latitude is approached. At latitudes greater than 75°, the minimum field along the line is no longer at the equator but near the null point.

A number of interesting consequences for trapped-particle studies emerge from these results. First, it should be very difficult to permanently trap particles along field lines above the critical latitude on the daylight side, because of the weak fields, long field lines, and high turbulence along parts of the field line. This conclusion would also apply to latitudes less than but close to the critical latitude on the day side. It might be expected, for instance, that no particles would be stably trapped at latitudes above the latitude at which the minimum field on the day side is no longer at the equator, i.e., 75° in this analysis. However, it would be theoretically possible for low-energy particles to become trapped in one of the two valleys of magnetic field near the null points on the day side.

The high-latitude cutoff has been confirmed by satellite studies of trapped particles. As expected, most of the radiation is trapped at low and middle latitudes. No trapped particles have been found over the poles. From Injun 1 data *O'Brien* [1963] has found a large diurnal variation in the high-latitude termination of trapped electrons. On the average this cutoff was at around 75° in local day and 69° in local night. The 75° cutoff would be reasonable in view of the closeness to the critical latitude. Computations are now under way to determine the drift paths of these particles by assuming conservation of the first two adiabatic invariants, to determine whether the drift to lower latitudes on the night side is sufficient to cause the observed reduction in the high-latitude cutoff there.

All observations so far have been based on calculations of field lines and field magnitudes in the noon-midnight meridian. This is the easiest situation to visualize, since the ϕ component of the field vanishes and the field lines are all

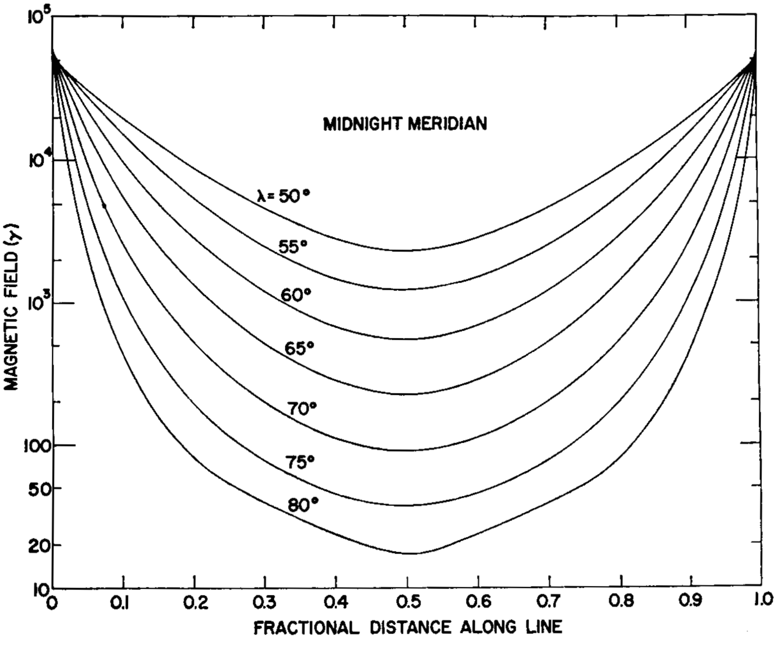

Fig. 6. Absolute magnitude of the field along lines on the midnight meridian.

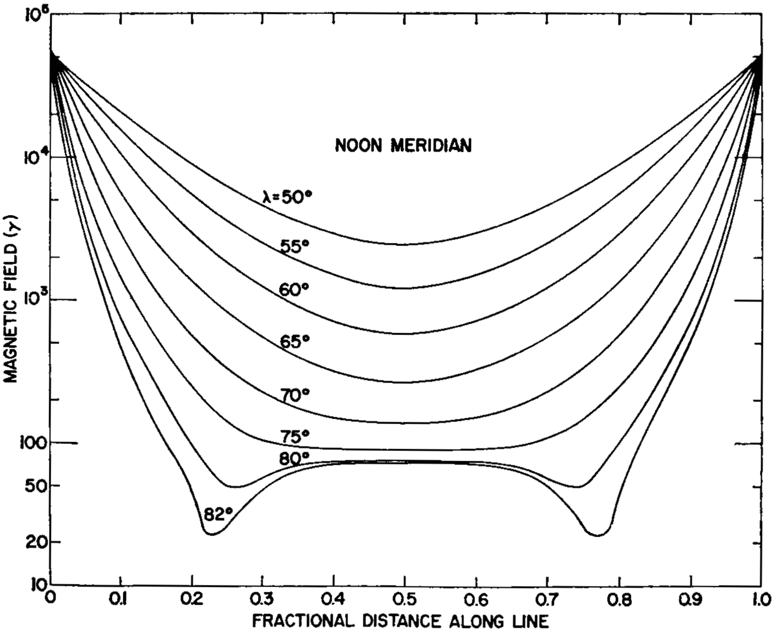

Fig. 7. Absolute magnitude of the field along lines in the noon meridian.

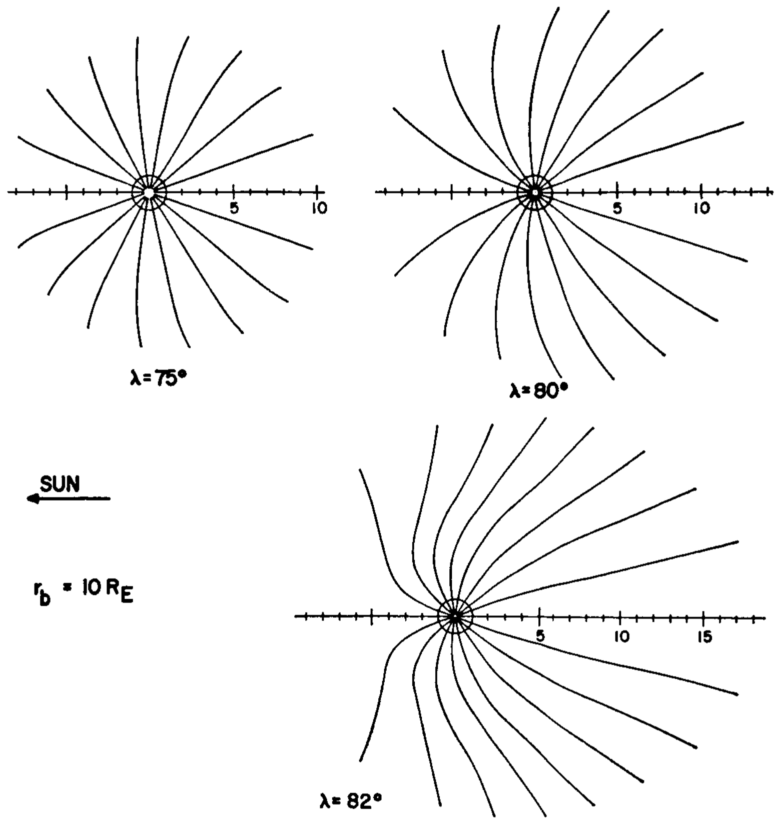

Fig. 8. View of the field lines from above the north pole. The lines are shown in 20° intervals of ϕ.

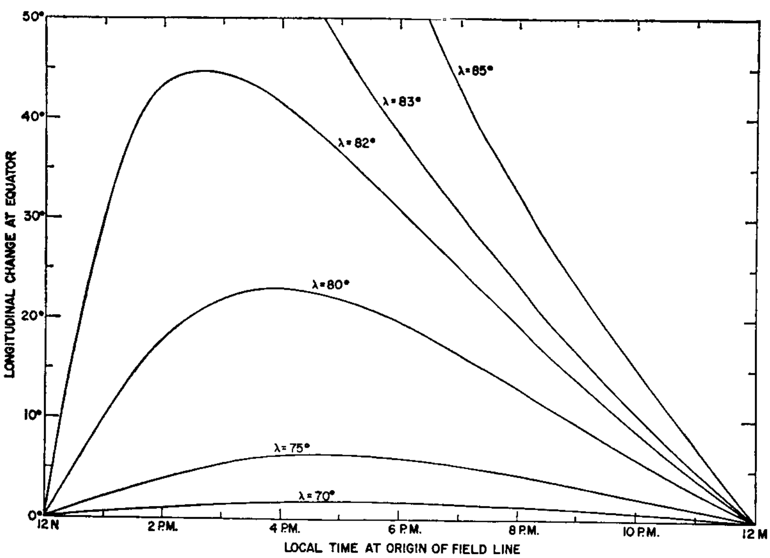

Fig. 9. Longitudinal change of the field lines at the equatorial crossing point. The effect is negligible at latitudes less than 70°.

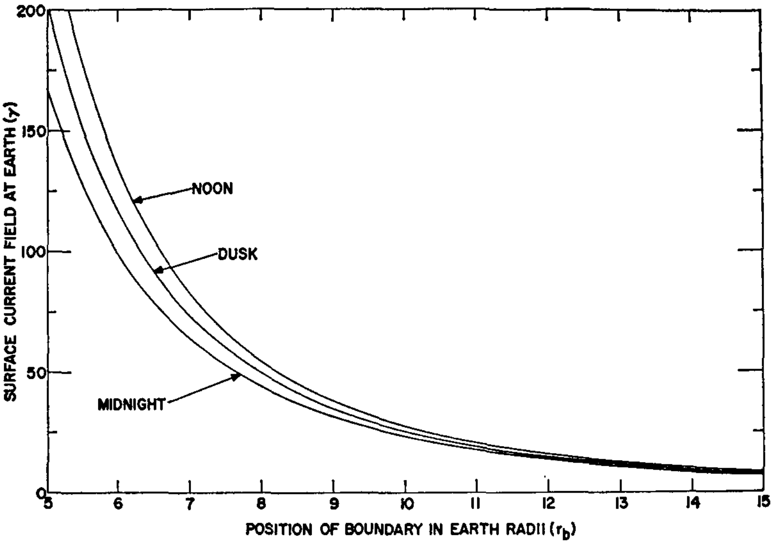

Fig. 10. Surface current field along the equator at the earth's surface as a function of r_b. From this, the increase in the field at the earth during a sudden commencement can be related to a change in the position of the boundary.

coplanar. As we move away from the noon meridian, the magnetosphere surface currents give rise to an azimuthal field component, which in general moves the equatorial crossing point of each field line away from the sun. The lines emanating from a given longitude are thus no longer coplanar, and it becomes more difficult to illustrate their behavior. One way of seeing this effect is to view the field lines from a position above the north pole. The lines then appear as projections upon the equatorial plane. Figure 8 gives three such views, each of which shows different longitudinal lines emerging from a given latitude. As expected, the azimuthal change is greatest for high-latitude lines. A line emerging from 80° latitude at a longitude 60° east of the noon meridian crosses the equator at 83° east longitude. The picture changes rapidly as the critical latitude is approached, and beyond this latitude all lines cross the equator within ±75° of the midnight meridian, regardless of the longitude from which they originated.

This same effect is shown graphically in Figure 9. Here the change in longitude at the equator is plotted as a function of the local time at the point where the lines emerge from the earth. Again the difference between the 82° lines and the 83° lines can be clearly seen.

Sudden Commencements of Magnetic Storms

The effect of sudden commencements can be calculated by using the present field description, ignoring transient variations (such as those caused by the propagation of hydromagnetic waves), and assuming that the entire magnetosphere adjusts at once to an increase in the intensity of the solar wind. The only effect is then to reduce all magnetosphere dimensions by a constant factor and increase the internal fields proportionately. Figure 10 shows the results of this calculation. Here the field along the geomagnetic equator at the earth's surface due to the magnetosphere surface currents is plotted as a function of r_b, the position of the boundary. If the boundary is at 10 earth radii, the surface current field is 27 gammas at noon, 25 gammas at 6 PM, and 23 gammas at midnight. A sudden increase in the solar wind causing the boundary to shrink to 6 earth radii would increase the field at the earth to around 115 gammas, producing an observed increase of 90 gammas in the north component of the field at the geomagnetic equator. The energy density of the solar wind would have to be increased by a factor of about 20 to produce this change.

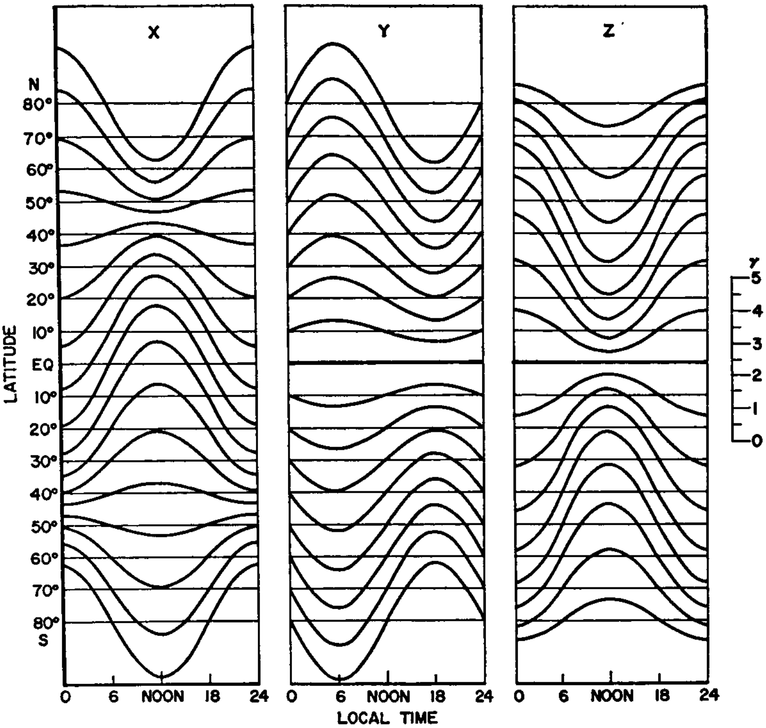

Fig. 11. Diurnal variations in the earth's field due to a steady solar wind. X, Y, and Z are the north, east, and downward vertical components of the field at the earth's surface.

An approximate analytic expression for the latitude dependence can be obtained from (10)–(12) by taking only the leading terms in the surface current field, those proportional to $\bar{g}_1^0$. These terms are independent of local time, and

$$\Delta X = \Delta B \cos \lambda \qquad (18)$$

$$\Delta Y = 0$$

$$\Delta Z = -\Delta B \sin \lambda \qquad (19)$$

where

ΔB (gammas)

$$= \frac{25{,}000}{r_{b_1}^{\ 3}}\left[\left(\frac{r_{b_1}}{r_{b_2}}\right)^3 - 1\right] \approx \frac{75{,}000}{r_{ave}^{\ 4}}\,\Delta_{r_b}$$

where r_{b_1} is the quiet-time position of the boundary, r_{b_2} is the new equilibrium position after the solar wind has increased, $r_{ave} = (r_{b_1} + r_{b_2})/2$, and $\Delta r_b = r_{b_1} - r_{b_2}$. Using (10$a$)–(12$a$), the field change may be expressed directly in terms of solar wind parameters:

$$\Delta B \text{ (gammas)} = 0.0305(\sqrt{n_2}v_2 - \sqrt{n_1}v_1)$$

with n in protons/cm^3 and v in km/sec.

It is clear from (18) and (19) that an increase in solar wind intensity will add to the north or X component of the earth's field, but reduce the earth's vertical component. This is true in both the northern and southern hemispheres. The effects of induced earth currents, however, have not been included.

This type of analysis cannot account for the main phase of magnetic storms, since the maximum *decrease* in the horizontal component would occur only if the solar wind were to vanish entirely. If the quiet-time boundary were at 10 earth radii the maximum decrease would be only about 25 gammas, whereas decreases of several hundred gammas are often observed. Thus, the current sources producing the main phase of magnetic storms cannot be those at the magnetosphere boundary.

The Contribution to S_q Fields by the Solar Wind

From the description of the distorted field in this paper, the daily variation of the three components of the magnetic field at the surface of the earth can be calculated at different latitudes. That is, X, Y, and Z of (10)–(12) can be calculated as a function of latitude λ and local time ϕ for $r = 1$. This is equivalent to calculating the portion of the S_q (solar quiet) field due to surface currents at the boundary of the magnetosphere.

The results are shown in Figure 11 for $r_b = 10$ earth radii. The variation shown here is almost entirely due to terms proportional to $\bar{g}_2^1$. The $\bar{g}_1^0$ terms are time independent, and higher-order terms are very small compared with the $\bar{g}_2^1$ terms at $r = 1$.

An approximate analytic expression for the daily variation produced by the solar wind may be obtained from (10)–(12) by keeping only terms proportional to $\bar{g}_2^1$:

$$X \text{ (gammas)} = -\frac{21{,}500}{r_b^4} \cos 2\lambda \cos \phi \qquad (20)$$

$$Y \text{ (gammas)} = \frac{21{,}500}{r_b^4} \sin \lambda \sin \phi \qquad (21)$$

$$Z \text{ (gammas)} = \frac{21{,}500}{r_b^4} \sin 2\lambda \cos \phi \qquad (22)$$

where the expressions are now in terms of the geomagnetic latitude λ instead of the colatitude θ.

Figure 11 is qualitatively very similar to that obtained from Chapman's spherical harmonic analysis of the earth's S_q field, described in *Chapman and Bartels* [1940, vol. 1, p. 215]. This is due to the fact that in both cases the $\bar{g}_2^1$ term is the leading term and is of the same sign. There are three important differences, however:

1. The observed S_q field shows typical variations of 20–40 gammas at midlatitudes, whereas the S_q field produced by the solar wind as calculated here gives a variation of only 2–3 gammas. The boundary would have to be brought down to 5–6 earth radii by the solar wind to produce changes as large as those actually observed. The quiet-time boundary has not been observed to be as close as this.

2. The observed daily changes are much greater during the hours of sunlight than during those of darkness. This is not predicted by the present analysis.

3. The present analysis does not predict the very large daily variations (100–200 gammas) observed at stations very near the magnetic equator, e.g., Huancayo, Peru.

Therefore, although the calculated daily variations produced by the solar wind are qualitatively similar to those actually observed, they differ in magnitude and in details. We conclude that the solar wind makes only a small contribution to the total S_q field. The major part is almost certainly due to current systems in the ionosphere together with the associated induced earth currents.

Discussion and Conclusions

There are a number of limitations in the accuracy of the field description used in this paper. First of all, the underlying assumptions—a steady solar wind, no magnetic field in the interplanetary region, specular reflection of the solar wind at the magnetosphere boundary, the absence of a shock wave or transition region, and perpendicular incidence upon a pure dipole colinear with the earth's axis of rotation—are not fully met. Second, contributions from external sources other than those at the magnetosphere boundary have not been considered. Third, a completely nonconducting magnetosphere has been assumed, whereas currents induced either in the earth's interior or in the conducting ionosphere would modify the situation.

Despite these limitations, we believe that the present picture is at least qualitatively correct. The fact that the earth is not purely dipolar would alter the details but not the main results. The presence of significant ring currents would change the field line picture, but there are indications that, at least during quiet times, the surface current fields are the dominant external influence in the magnetosphere. Perhaps the most serious limitation is the assumption that the solar wind is perpendicular to the dipole. The dipole axis can be tilted as much as 35° either toward or away from the sun, depending on time of day and time of year. In addition, the solar wind velocity vector may not be parallel to the earth-sun line. These conditions would remove the north-south and east-west symmetry conditions inherent in the present description.

Several conclusions stand out in the present analysis. It is possible to describe the earth's distorted field with reasonable accuracy using only a few numerical coefficients for the surface current terms. With this description, the field lines in the noon-meridian plane exhibit some unusual characteristics as a certain critical geomagnetic latitude is approached, which varies from 80° to 85°, depending on the solar wind intensity. For lines emanating from points near to but less than this latitude, the minimum value of the magnitude of the field along the field line occurs at a high latitude near the null point rather than at the equatorial crossing point, as would ordinarily be expected. Lines emerging beyond the critical latitude are bent over the north and south poles and cross the equator along the midnight meridian. Lines emerging near the critical latitude but away from the noon meridian tend to be pushed back toward the dark side. At lower latitudes the presence of the solar wind compresses the field lines on both the dark side and the light side, although the effect is larger on the light side. The effect of the solar wind is significant only upon lines emerging at latitudes greater than 60°. A number of consequences of this over-all field line behavior upon trapped radiation and geomagnetic activity have been examined.

The field description permits the calculation of the effects at the earth's surface due to an increase in the solar wind, as is observed during a sudden commencement. In both hemispheres the earth's horizontal component is increased but the magnitude of the vertical component is decreased. The diurnal changes in the components of the earth's field due to rotation under a steady magnetosphere have been calculated; the resulting variations were found to be small compared with the observed S_q fields, indicating that these fields are not primarily due to magnetosphere surface currents.

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References

Akasofu, S.-I., The main phase of magnetic storms and the ring current, *Space Sci. Rev., 2,* 91–135, 1963.

Beard, D. B., and E. B. Jenkins, The magnetic effects of magnetosphere surface currents, *J. Geophys. Res., 67,* 3361–3367, 1962.

Cahill, L. J., and P. G. Amazeen, The boundary of the geomagnetic field, *J. Geophys. Res., 68,* 1835–1843, 1963.

Chapman, S., and J. Bartels, *Geomagnetism,* Clarendon Press, Oxford, 1940.

Heppner, J. P., N. F. Ness, C. S. Scearce, and T. L. Skillman, Explorer 10 magnetic field measurements, *J. Geophys. Res., 68,* 1–46, 1963.

Hones, E. W., Motions of charged particles trapped in the earth's magnetosphere, *J. Geophys. Res., 68,* 1209–1219, 1963.

Lebeau, A., and R. Schlich, Sur une propriété de l'activité magnétique diurne dans les régions de haute latitude (Stations Charcot et Dumont d'Urville), *Compt. Rend., 254,* 3014, 1962.

Mead, G. D., and D. B. Beard, Shape of the geomagnetic field solar wind boundary, *J. Geophys. Res., 69*(7), 1964.

Midgley, J. E., Perturbation of the geomagnetic field—a spherical harmonic expansion, *J. Geophys. Res., 69*(7), 1964.

Midgley, J. E., and L. Davis, Jr., Calculation by a moment technique of the perturbation of the geomagnetic field by the solar wind, *J. Geophys. Res., 68,* 5111–5123, 1963.

O'Brien, B. J., A large diurnal variation of the geomagnetically trapped radiation, *J. Geophys. Res., 68,* 989–995, 1963.

Wescott, E. M., and K. B. Mather, Diurnal effects in magnetic conjugacy at very high latitude, *Nature, 197,* 1259, 1963.

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